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"I hold every man a debtor to his profession, from the which as men of course do seek to receive countenance and profit, so ought they of duty to endeavour themselves by way of amends to be a help and ornament thereunto."—BACON.

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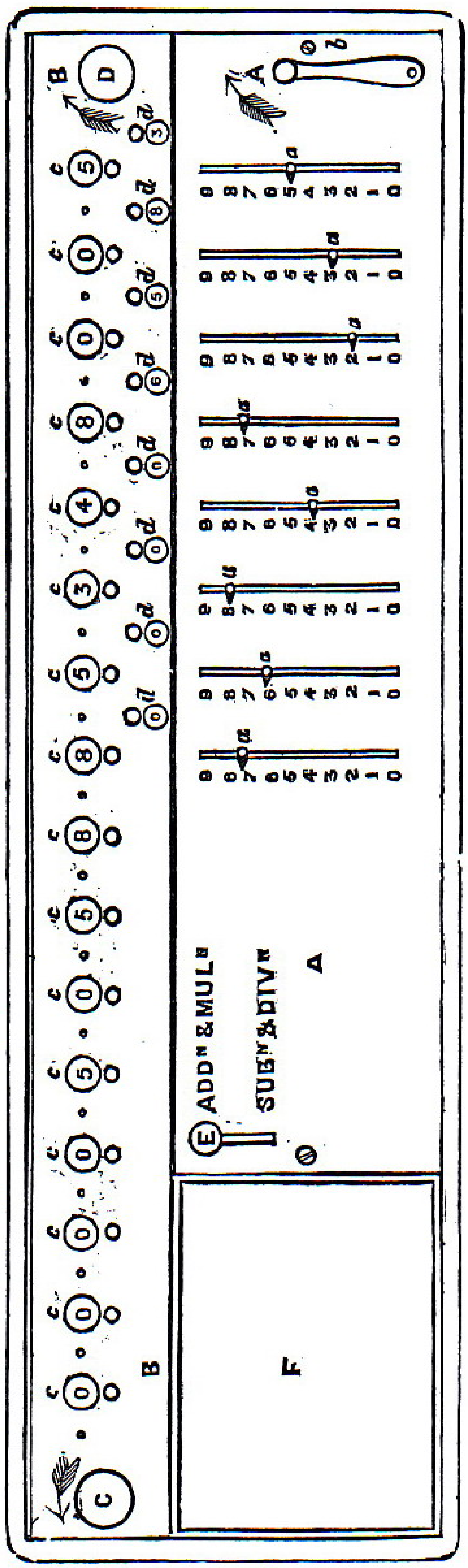
On the Use of M. Thomas de Colmar's Arithmometer in Actuarial and other Computations. *By* MAJOR-GENERAL HANNYNGTON, *Associate Member of the Institute of Actuaries.*

[Read before the Institute, 27th February, 1871.]

THE accompanying illustration gives a top view of the instrument. The following description is taken from the *Engineer*, 20th May, 1870.



It is constructed chiefly of a brass plate A A furnished with eight slots as shown; directly under these slots are mounted eight drums, each having nine elongated cog-teeth of successively decreasing length; over each drum, and between it and the slot, is mounted a square shaft on which slides a pinion wheel, so as to catch any number of teeth on the drum. Each of these pinion wheels is moved by a button *a*, of which there is one in each slot, the figures at the sides of the slots showing the proper position of each button *a* for any work to be performed by the instrument, so that not the least trouble is encountered in arriving at the result. The cogged drums gear by bevel wheels with a long horizontal shaft which is also in gear with the vertical shaft moved by the handle *b*, by which the instrument is worked. B B is a moveable brass plate, which can turn and slide on a round bar hinge at the back; in this plate there are sixteen holes, *c*, under each of which is a moveable disk numbered from 0 to 9, and arranged so that any one figure of each disk may be brought under its corresponding hole *c*. These disks have bevel wheels, which gear with bevel wheels on the before-mentioned square shafts. The moveable plate B B is also furnished with the holes *d*, having disks numbered from 0 to 9 underneath, and are for showing the number of turns of the handle, giving by this means the quotient in division and showing the multiplier in multiplication. The knobs C and D are for bringing the figures under the holes *c* and *d* respectively to zero before commencing an operation, and the knob E is for setting the instrument to work addition and multiplication, or subtraction and division. F is a small plate for memorandums. If the knob

E be placed at addition, each turn of the handle will carry the figures marked by the buttons *a* under the indicator holes *c*, while if the knob be placed at subtraction it will subtract from the figures under the holes *c* the number marked by the buttons *a*.

The instructions given with the machine are sufficient for the performance of the elementary operations, but it will nevertheless be convenient in this paper to explain these operations anew. A reference to the diagram will render what follows intelligible.

The fixed brass plate A A may be called the Face Register or *Face,* on which numbers expressed by eight or fewer figures may be indicated by the moveable *markers a.* When we desire that any given number should be so indicated, the direction will be to *set on* that number. The moveable plate BB we shall call the Sliding Register or *Slide;* and when we desire that a number be transferred from the Face to the Slide, the direction will be to *put up* that number. In the diagram the number 76847235 is "set on" and the number 505885348005 is "put up," while the number 6583 appears in the quotient holes. The figures in the upper holes show the product of the first and last of these numbers. Sometimes it will be convenient to put up numbers by hand, which may be done by turning the *Studs* which are under the *figure holes c* (sixteen in number in the diagram). These studs are placed on the axes of the *figure disks,* which disks turn in either direction, and have the numerals 0 to 9 so ranged on them as to appear, one at a time, in the figure holes. A smaller set of holes, *d,* studs, and disks (in the diagram eight in number) are placed below the others on the right-hand half of the slide; these may be called the "*quotient*" *holes, studs,* and *disks,* being, as will appear, serviceable chiefly in the operation of division. The small holes between the figure holes, *c,* serve to hold an ivory pin to mark, when requisite, the decimal point. By turning the milled head C towards the left hand, and the head D towards the right hand, the figures in the holes *c* and *d* respectively will become zero; these heads may therefore be called *Effacers.* Before commencing any operation, it is necessary to see that all significant figures have been effaced. The button E, which slides along the slit shown in the diagram, may be called the *Regulator*, because, as marked on the Face, its position determines the action of the instrument. The *Winch, b,* turned by hand, always in one direction, to the right, gives motion to the mechanism. One turn "puts up" the number that has been "set on" the face; and if this number be not changed, another turn will put it up again, and so continually. Under the winch lies a pin, not shown in the diagram. This may be called the *Stopping pin*, at which the winch should always be stopped. When the winch is not at the stopping pin, the regulator cannot be moved. Attached to the markers, under the face, is a spring, to keep them, when in use, in their proper position. F is a small plate of ground glass to serve as a writing tablet.

We may now commence numerical operations. They will be adapted to an eight-figure machine.

Addition.

Place the regulator at addition; set on the several addends in succession, giving one turn of the winch for each.

Example.—Find the sum of 94765, 34287, and 52931.

Set on, and put up		94765,	result	94765
,,	,,	34287	,,	129052
,,	,,	52931	,,	181983

In this way any series of numbers, each consisting of not more than eight figures may be summed, so long as the sum does not exceed 9999999999. A further example will be given, in showing how to extend the power of the machine.

The operation of addition is rather tedious; but when the sum of many numbers is required, the result is safe.

Subtraction.

Place the regulator at addition; set on the minuend, and one turn of the winch will put it up; or the minuend may be put up on the slide by hand, which is often more convenient. Place the regulator at subtraction; set on the subtrahend and give one turn of the winch.

Example.—From 181983 take 94765.

Put up the minuend 181983; set on the subtrahend 94765. Placing the regulator at subtraction, give one turn of the winch, and the result, 87218, will appear on the slide.

A series of addends and subtrahends, arranged in any order of succession, may be summed by attending to the signs, and placing the regulator accordingly. In such case, the occurrence of an intermediate negative value will not embarrass the action of the machine, or interfere with the exhibition on the slide of a final positive result. But if the sum of the subtractive quantities exceed that of the additive quantities, the result will show one or more superfluous figures, nines, on the left of the slide. These nines must be rejected, and the complements of the other figures be taken. Thus, if the preceding example be inverted, the result will show 9999912782, which represents −87218.

Multiplication.

Place the regulator at addition and set on the multiplicand. Give as many turns of the winch as are indicated by the digit in the units, or in the lowest place. This digit will now appear in the quotient hole on the right, and a corresponding product in the larger figure holes. Now draw out the slide towards the right hand one or more places, as indicated by the place of the next significant figure of the multiplier reckoned from the extreme right, and give the number of turns indicated by that figure, which will then appear in its proper quotient hole. The product of the multiplicand and the two figures of the multiplier will be seen in the large figure holes. Proceed similarly, digit by digit, until all the digits of the multiplier appear in their order in the quotient holes. The complete product of the two factors will then be exhibited in the figure holes of the slide. It will be convenient to designate the factor that is set on the machine as the *in* factor, and the other, which is ultimately seen in the quotient holes, as the *out* factor.

As an example, let us multiply 42967 by 3752.

Place the regulator at multiplication; set on 42967.

The units figure being 2, give two turns, the result is 85934
Draw out the slide one place;
The next figure being 5, give five turns, the result is 2234284
Draw out the slide one place;
" 7, give seven turns, the result is 32311184
Draw out the slide one place;
" 3, give three turns, the result is 161212184

The operation is now complete, and the careful worker will notice that the out factor 3752 appears in the quotient holes, and shows that the right number of turns has been made.

Division.

Draw out the slide to its full extent, *i.e.*, so that eight holes only are left on the face. Put up the dividend. Be careful before commencing the division to have the quotient disks effaced. Set on the divisor, place the regulator at division, and turn until the figures on the left of the slide become less than those under them on the face. Bring in the slide one or more places to the left, as may be requisite to make the figures remaining thereon exceed in arithmetical value those under them on the face, and turn as before. Repeat the process until the slide be wholly brought home, or until the dividend be exhausted. The quotient will now appear in the quotient disks.

As an example, we may find $\frac{161212184}{42967}=3752$, and the process will exhibit in reverse order the same figures as the example of multiplication above given.

The four arithmetical rules having been thus exemplified within the ordinary limits of the machine, it is proper to show how these limits may be extended.

The addition or subtraction of large numbers can be performed in parts, and to this method there are practically no limits. A single example will suffice.

3579	62176543
5962	75311739
7346	16753761
2517	93665716

Taking eight places from the units inclusive, the sum 247907759 appears on the slide; then drawing out the slide eight places, the four places on the left hand of the vertical line will be summed in their proper position, and the total, 1940647907759, will be found.

As to multiplication, some detail is necessary.

The machine to which these illustrations refer, is an eight-figure machine, giving products up to sixteen places of figures, which is the full extent of the slide. But with intermediate record by the operator in the manner shown below, the power of the machine can be virtually doubled, and a product containing thirty-two places of figures be readily obtained. For if each factor be severed into parts of eight figures each, and these parts be denoted by the letters *a, b, c, d,* we have

$$(a+b)(c+d)=ac+ad+bc+bd$$

whereby not only the partial products but also their local positions are indicated, so that they may be written in due order.

For example,

$$\underbrace{12345678}_{a}\ \underbrace{87654321}_{b}\times\underbrace{86427531}_{c}\ \underbrace{13572468}_{d}$$

```
ac=1067006468061018
ad=        0167561319593304
bc=        7575746545511451
bd=                1189685466834228
   --------------------------------
   10670065454940967700160966834228
```

Rarely indeed can such a product as this be required, but often there is need for partial products to a considerable number of places, and that these should be obtained without intermediate record. The extent to which this is possible is $n+\frac{1}{2}n$ places, true in the last place, n being the number of places on the face of the machine. Hence, an eight-figure machine will give partial products of twelve figures, true in the last place. The nature of the process is shown above, and the practical rule is as follows, for twelve figures by twelve figures:—

Divide each of the factors into parts, of eight figures and four figures, reckoned from the highest place, and call $(a+b)$ the *in* factor; set on the first eight places of the *in* factor, multiply by first eight places of the *out* factor; set on the first four places of the *in* factor, multiply by the last four places of the *out* factor; set on the last four places of the *in* factor, multiply by the first four places of the *out* factor.

The work is now complete, and is true in the twelfth figure, reckoned from the highest place.

We repeat the former example, cut down to twelve figures,

$$\underbrace{12345678}_{a}\ \underbrace{8765}_{b} \times \underbrace{86427531}_{c}\ \underbrace{1357}_{d}$$

The result is

$$106700654549\,1451$$

of which twelve places are true.

Division is less tractable than multiplication, but so long as the divisor contains no more than eight figures the dividend and the quotient may be unlimited, for the remainders can be transferred to the left or highest place on the slide, the partial quotient be recorded, then effaced, and the operation be carried on to any extent. But a divisor of more than eight figures offers difficulties that can perhaps best be surmounted by decomposing it into two or more factors, and then working by successive divisions. This branch of the subject may be left to inquiring students.

Among the powers of the machine may to some extent be included that of the difference engine; for a second difference can often be supplied by the operator. For instance, a table of square numbers having the second difference constant, requires merely that the operator should continually add 2 to the difference on the face of the machine. Thus in fact any quadratic form could easily be tabulated. Hence, also, two or more operators working together on separate machines, might compute tables requiring differences of the third or higher orders. Such an application of these machines might have important uses.

To the actuary these machines may be very helpful, and I shall therefore offer a few examples based on life contingencies. The summation of Columns N_x, S_x and the like, where each successive value has to be exhibited, may be very conveniently effected.

Detached operations, such as $D_x = l_x v^x$, do not require explanation. To obtain the values directly, and without having to record the intermediate steps, is an obvious advantage.

When $D_{x.y} = l_x v^x . l_y$ for all combinations of x and y is required, the machine affords peculiar aid.

Since each value of D_x has to be combined with each value of l_y, any given value of D_x being set on as an *in* factor may be multiplied by successive values of l_y, which operation can be made continuous by either adding or subtracting $D_x d_y$. It is immaterial whether we begin with the youngest or oldest age, for the machine performs addition or subtraction with equal facility.

The type of the calculation is either

$$D_x l_y + D_x d_{y+1} + D_x d_{y+2} + \ldots .$$

or

$$D_x l_y - D_x d_{y-1} - D_x d_{y-2} - \ldots .$$

and the successive values are to be entered in their proper columns as they arise. As a check, $D_{x.y}$ for each tenth year of difference of ages should be directly computed, and the agreement of these with the continuous process will prove the work.

The data for Carlisle Table, 3 per cent., are as follows, to a sufficient extent for our example:—

x	l_x	d_x	v^x	D_x
104	1	1	·04623050	0·04623050
103	3	2	·04761742	0·14285226
102	5	2	·04904594	0·24522970
101	7	2	·05051732	0·35362124
100	9	2	·05203284	0·46829556
99	11	2	·05359382	0·59853202
98	14	3	·05520164	0·77282296
97	18	4	·05685769	1·02343842
96	23	5	·05856342	1·34695866
95	30	7	·06032032	1·80960960
94	40	10	·06212993	2·48519720
93	54	14	·06399383	3·45566682
92	75	21	·06591364	4·94352300
91	105	30	·06789105	7·12856025
90	142	37	·06992778	9·92974476

In this l_x and l_y are taken from the same life table, but they may, and often should be, taken from different life tables, as for instance when the lives represent husband and wife, or father and daughter.

The operation will now be,—Set on D_{104}=·04623050 (this remains on the face). Multiply by $l_{104}=1$; record the result, ·04628050, in Column 0, and let it remain on the slide. Efface the out factor. Multiply by $d_{103}=2$; record the result, 0·13869150, in Column 1, and let it remain on the slide. Efface the out factor. Multiply by $d_{102}=2$; record the result, 0·23115250, in Column 2, and let it remain on the slide. Efface the out factor. Multiply by $d_{101}=2$, and proceed as already shown.

The speed and certainty of this method will be appreciated by those who have had experience in the construction of commutation tables.

The effacing of the out factors is not a part of the numerical details, but it is a precaution against error that will not be neglected by careful workers.

The following values of $D_{x.y}$ were computed on the machine (at full power) by the foregoing process. They will serve as a guide to those who may desire to travel in this path. The decimals are here recorded to an extent not required in practice:—

$x-y=$	0	1	2	3	4	5
$x=104$	0·0462305	0·1386915	0·2311525	0·3236135	0·4160745	0·5085355
„ 103	0·4285568	0·7142613	0·9999658	1·2856703	1·5713749	1·9999316
„ 102	1·2261485	1·7166079	2·2070673	2·6975267	3·4332158	4·4141346
„ 101	2·4753487	3·1825912	3·8898336	4·9506974	6·3651823	8·1332885
„ 100	4·2146600	5·1512512	6·5561378	8·4293201	10·7707979	14·0488668
„ 99	6·4848522	8·2534483	10·6115764	13·5592365	17·6859606	23·5812808
„ 98	10·8195214	13·9108133	17·7749281	23·1846888	30·9129184	41·7324398
„ 97	18·4218912	23·5390832	30·7031520	40·9375360	55·2656736	76·7578800
„ 96	30·9800501	40·4087610	53·8783480	72·7357698	101·0219025	141·4306635
„ 95	54·2882880	72·3843840	97·7189184	135·7207200	190·0090080	256·9645632

NOTE.—The first entries in Columns 0, 1, and 2 are found as shown in the foregoing explanation, and in like manner all the values are computed in lateral order from the initial term in Column 0. The vertical columns are $D_{x.y}$ from the oldest ages downwards, for differences of age noted in the headings. They might, with equal facility, be formed from the younger ages, as is usual in the construction of Column D.

In a similar way survivorship assurances may be computed, but an example is unnecessary. What is here shown brings out one of the most useful powers of the machine, namely that of giving the *sum* of a series of *products* without exhibiting the several quantities.

A simple and striking example of continuous calculation is afforded in the construction of temporary annuities. If we set on the reciprocal of D_x at any age x, and then multiply this by the successive values of D for the higher ages, never altering what appears on the slide, our end will be attained. The form is expressed thus,

$$\frac{1}{D_x}(D_{x+1}+D_{x+2}+D_{x+2}\ldots\ldots+D_{x+n})={}_{n)}a_{x+n}$$

The successive values as they arise on the slide must be recorded.

The formation of tables having a constant difference requires no more than the setting on of an initial value, and thenceforth each turn of the winch gives a new term. Thus, by placing 1 in a proper position on the slide, and $\frac{1-v}{100}$ on the face, and setting the regulator to subtraction, we might grind out tables for the conversion of annuities into assurances more quickly than the values could be transcribed.

To pursue the subject further would be tedious. I will therefore conclude with two examples in a different branch of science. They are, I think, remarkable and important.

1. Given a, b, c, three sides of a spherical triangle to find an angle—suppose the angle A.

2. Given two sides b, c, and the included angle A of a spherical triangle to find the side a.

The formulas that we shall use are

$$\sin^2\tfrac{1}{2}A=\frac{\sin^2\frac{1}{2}a-\sin^2\frac{1}{2}(b-c)}{\sin^2\frac{1}{2}(b+c)-\sin^2\frac{1}{2}(b-c)}\quad\ldots\ldots\quad(1)$$

$$\sin^2\tfrac{1}{2}a=\{\sin^2\tfrac{1}{2}(b+c)-\sin^2\tfrac{1}{2}(b-c)\}\sin^2\tfrac{1}{2}A+\sin^2\tfrac{1}{2}(b-c)\quad(2)$$

It appears from these formulas that the data are all of one name—that is, the squares of the sines of half arcs.

Let the sides and angle be as follows:—

	θ °	′	″	$\text{Sin}^2\frac{1}{2}\theta$
a =	79	45	39	0·4111212
b =	73	45	51	·3602051
c =	39	53	20	·1163552
A =	93	25	25	·5298593
$(b+c)$=	113	39	11	·7005986
$(b-c)$=	33	52	31	·0848735

Now, to find $\sin^2\frac{1}{2}A$, we have

$$\frac{\cdot4111212-\cdot0848735}{\cdot7005986-\cdot0848735}=\frac{\cdot3262477}{\cdot6157251}=\cdot5298593=\sin^2\tfrac{1}{2}A\,;$$

and to find $\sin^2\frac{1}{2}a$,

$$\begin{array}{r} \cdot 7005986 \\ -\cdot 0848735 \\ \hline \cdot 6157251 \end{array} \times \cdot 5298593 = \begin{array}{r} \cdot 3262477 \\ +\cdot 0848735 \\ \hline \cdot 4111212 \end{array} = \sin^2\tfrac{1}{2}a$$

The simplicity of these operations is evident; and as these two problems comprise the whole practice of nautical spherics, they are in this form worthy of especial notice.

A table of $\sin^2\frac{1}{2}\theta$ for every tenth second of space, with suitable formulas, was published in 1805 by James Andrew, M.A. This is one of the most valuable aids ever offered to navigators, and yet, so far as I know, it has been utterly neglected by them. Conjointly with the arithmometer it would smooth all the difficulties of lunars, and double altitudes, and of great circle sailing; and if this mention of it be somewhat out of place, it is nevertheless well deserved. Extensive tables of natural versed sines exist, and they afford similar but not in all respects the same advantage.

I have noticed that tables quadratic in form may be computed on the machine. A conspicuous instance may be mentioned. Sir John Macneill's Earthwork Tables are purely quadratic, and they might thus, with comparatively small labour, be recomputed and extended. There are many other tables having second differences that change but slowly; these also are within the range of the machine.

In conclusion, I may say that, having had considerable experience in actuarial computations, I have never found the machine fail to afford help; though it may happen that the right process is not always that indicated by the seemingly most appropriate formula. The machine asks for peculiar methods, and such as are not easily to be described. Herein there is room for skill and intelligence, so that hand and head may work together with mutual advantage. I will only add, that I stand here, not as a teacher but as a student and fellow worker with the members of this Institute, whom I have the honour to address.

On Mechanical Aids to Calculation. A Lecture to the Actuarial Society of Edinburgh. By EDWARD SANG, F.R.S.E., *Fellow of the Faculty of Actuaries in Scotland.*

IN every branch of business, even in the very rudest stage of barter, men have to count. A skin is exchanged for so many cowries, a rifle for so many skins. Do what we will, go where we will, the necessity for counting meets us. Yet, though numbers be essential to all our operations, we do not easily form an idea of large numbers. Thus in no language do the separate names for numbers reach to twenty. In one, I believe, the numeration goes as far as fifteen; in the northern languages of Europe it reaches to twelve, but in most languages it stops short at ten.

More advanced numbers are held as composed of so many tens and so many units; they are named accordingly, as in English thirty-seven, meaning three tens and seven. In the Greek, Latin, and modern European languages, the names of the multiples of ten are derived from the names of the units; in some Asiatic languages the first five of them have independent names.

When we arrive at ten tens a new name, as *hundred* in English, is introduced, and we reckon in hundreds up to ten hundreds, which also receives a special name, with us *thousand;* and here, with only one exception that I know of, the special nomenclature ceases; all larger numbers being named by compounding the names of inferior numbers.

In all this we see the use of helps to counting. Were numbers to receive independent names, the stock of words would be exhausted, the memory would fail to keep them in order, mistakes would be frequent. But the contrivance of articulate numeration introduces ease, clearness, and certainty.

[The above remarks appear to us extremely suggestive and worthy of further development. As regards the separate names of the numbers, however, it appears to us that the English names, *twenty*, *million*, *trillion*, *quadrillion*, must be considered as separate independent names. For it cannot be said that either *twenty* or *million*, or either of the others, is formed by compounding the (English) names of inferior numbers.

The help to counting furnished by a simple and systematic method of naming numbers, seems to have been carried further in English than in most other modern languages. Thus, as compared with French, the old-fashioned numeration so familiar to us in the English Bible, of *threescore*, *threescore and ten*, *fourscore*, &c., has entirely given way to *sixty*, *seventy*, *eighty*, &c., while in French *septante*, *octante*, *nonante*, are to this day provincialisms, and the words in common use are *soixante-dix*, *quatre-vingts*, *quatre-vingts-dix*. The method of numeration in Danish is still more clumsy. For *sixty* and *eighty* the names are *tresindstyve* and *firsindstyve*, which correspond to *threescore* and *fourscore*; but for *fifty*, *seventy*, and *ninety*, the names are *halvtresindstyve*, *halvfirsindstyve*, and *halvfemsindstyve*, which will correspond to *half* (-way towards) *threescore fourscore*, *fivescore*.

In comparing English with German, the idioms of the two languages closely coincide as regards the above points; but when we come to join units with tens, as in reading out the number 4321, the Englishman reads *four thousand three hundred* and *twenty-one*, while the German says *vier tausend drei hundert ein* und *zwanzig*, an inversion which must slightly increase the difficulty of counting, and which, we are informed, does in practice frequently cause mistakes in setting down the results of a multiplication.—Ed. *J. I. A.*]

As in the naming, so in the marking of numbers recourse was had to various contrivances for expediting the operations. The simple but prolix plan of putting down a counter for each article in barter, was soon improved by using a second kind of counter for each *ten*, or *dozen*, or *score* as the case might be. Thus in the very rudest kind of numeral notation which yet subsists, that used by the Romans, a stroke was made for each unit, and, to break the monotony, a cross stroke was used at the fifth, thus gradually giving rise to the use of the mark V for five and X for two fives or ten. The Arabs, however, adopted a more ingenious plan by using the first ten letters of their alphabet for the ten units; the second ten for the tens, and the third set of ten for the hundreds. A contrivance adopted by the Greeks, who, to supplement their defective alphabet, introduced some arbitrary marks. In India, however, the great improvement was made of using nine arbitrary marks for the nine units, and of indicating the tens, hundreds, and thousands by a change of position. This Hindi Rakkam was adopted by the Arabian arithmeticians, and by them introduced into Southern Europe; for which reason they are commonly but erroneously called by us Arabic numerals.

This Hindi contrivance, in daily use by all of us, is in reality a geometrical aid to counting: it takes a mechanical form in the shape of the swan-pan, or of the wire frame with strung beads.

When we mark down the successive numbers, the continual recurrence of the digits, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 0, 1, 2, introduces the idea of circulation; and so leads us to divide the circumference of a wheel into ten parts and to write the numerals in them. In this way, by turning the wheel one step at a time and always in one direction we have the successive units. And here we have the first element in machine-counting.

By arranging a second wheel for the tens, and so placing it as that at each turn of the unit's wheel it shall be moved one step, we are enabled to count to one hundred; and by continuing this arrangement we can carry the numeration to any desired extent. Such is the construction of ordinary counting-machinery. In some of these machines the movements are continuously connected by means of toothed wheels, as in the cases of clock-work and of the ordinary gas-meter index. In others the motions are by jerks; the ten's wheel remains at rest until the units are passing from 9 to 0, at which time the tens are advanced one step; and the indication changes from say 69 to 70. Some of these latter machines are so contrived as that the wheels are locked until the carrying take place. Such are the machines used for registering the number of operations performed by the bank-note printing press. From their very construction continuously connected machines are always locked.

Such counting machines are of great use in many situations, as at loading and landing wharves, in large warehouses, at turnpike gates and in general wherever extensive tale has to be made, or wherever a check is required upon operations. They receive various forms according to the purpose for which they are intended; and by modifications and extensions they become the calculating machines to which our attention is principally to be directed.

As soon as we attempt to use a counting machine we find that there must be some provision for setting it to zero, or to whatever may be the number from which we wish to begin. This provision is made in two ways. In one the index is fitted stiffly, not permanently, to its axis, so that it may be turned round whenever a sufficient pressure is applied to it. We have a familiar example of this in our clocks and watches: without it we should not be able to correct the error of the time-keeper. In the other way the circumference of the index wheel is notched and a knuckled holder enters into the notch, being kept there by a spring. By this arrangement the wheel cannot be turned either backwards or forwards without raising the holder, so that it is kept steadily in its place until a pressure be applied sufficient to overcome the elasticity of the spring. Moreover, as soon as the tooth of the wheel is brought past the angle of the knuckle, the index is thrown forward or backward to the next number. In such a case the index wheels are independent of each other; but complete independence would not subserve our purpose; wherefore a provision is made to cause an advance of unit on any one index when the index below passes from 9 to 0.

An instrument arranged in this way may be set to any initial number and may be used at once for counting. But it is now capable of much more extensive use.

If I turn the unit's index forward *seven* steps I shall add *seven* to the indication, for should the units overpass the 9, as say from 8 to 5, the ten's index would be brought forward one step by means of the carrier. If I now turn the ten's index *four* steps forward, I shall add *forty* to the indication, that is to say, I shall have added *forty-seven* to the previously recorded number. In this way the instrument has become an *addition machine;* its action contains all that is essential to this operation. It is also a subtraction-instrument; the very movements which perform addition may be made to perform subtraction, and even at the same time. All that is needed is to inscribe on the indices numbers in the inverse order.

Various contrivances have been made for the purpose of facilitating the addition; in all of these the object aimed at is to have the addend set upon the machine, and the summation performed by the motion of the instrument, generally by one turn of the handle. Much ingenuity has been expended on these contrivances, and considerable apparent diversity of arrangement has been the result; yet the essential characters of the action are the same in all.

We desire, for example, to bring the ten's index forward four steps by one action. For this purpose we may arrange that a hook or propeller shall have a stroke of four divisions; or we may cause four teeth of some wheel to act upon the index: both of these methods are in use; the latter is seen in Thomas' machine; the former in the striking part of our common house clocks, to which I shall have occasion afterwards to revert.

In Thomas' machine there is placed a cylinder to make a complete turn at each operation; on its surface there are fixed nine parallel bars, or as we may call them elongated teeth, which are to work the teeth of the counting wheel: the lengths of these bars are equi-different, so that at one place of the cylinder the whole *nine* are ready to act, at the next place *eight,* and so on. The counting wheel is made to slip longitudinally upon a squared axis, so that we can bring it opposite to any desired part of the cylinder.

If we bring the counting wheel so as to be acted on by four bars, one turn of the cylinder will cause an addition of *four,* and each succeeding turn will cause a new addition of *four.*

Now for each of the indices, that is for units, tens, hundreds, and so on, we have a special cylinder, and therefore if we set the counting wheels to specified digits, and turn each of the cylinders once round (for the present we shall suppose once in succession) we shall have added the specified number to the previous indication.

If we do this for one cylinder after another some considerable time will be required. The contrivance for economising that time is exceedingly ingenious in Thomas' machine. He causes all the cylinders to be actuated at once by the working handle, so that all the additions go on nearly at once. This, however, would cause confusion in the carrying. He therefore makes the index wheels entirely independent of each other, and arranges the carrying in another way. On each index wheel there is fixed a stud to come in contact with a lever whenever the indication passes from 9 to 0, and so to push this lever aside; and a detent is provided to keep this lever back after the stud has passed onwards. This lever brings into action the carrier fixed on the axis, and this carrier only acts after the addition by the bars has been completed. Thus by arranging the cylinders one tooth in arrear at each step, the carrying from one rank to another is completely effected, and the whole addition completed by one turn of the handle.

By help of this machine, then, we can perform addition, and by turning the handle repeatedly, we can perform successive additions, and so form an equi-different progression.

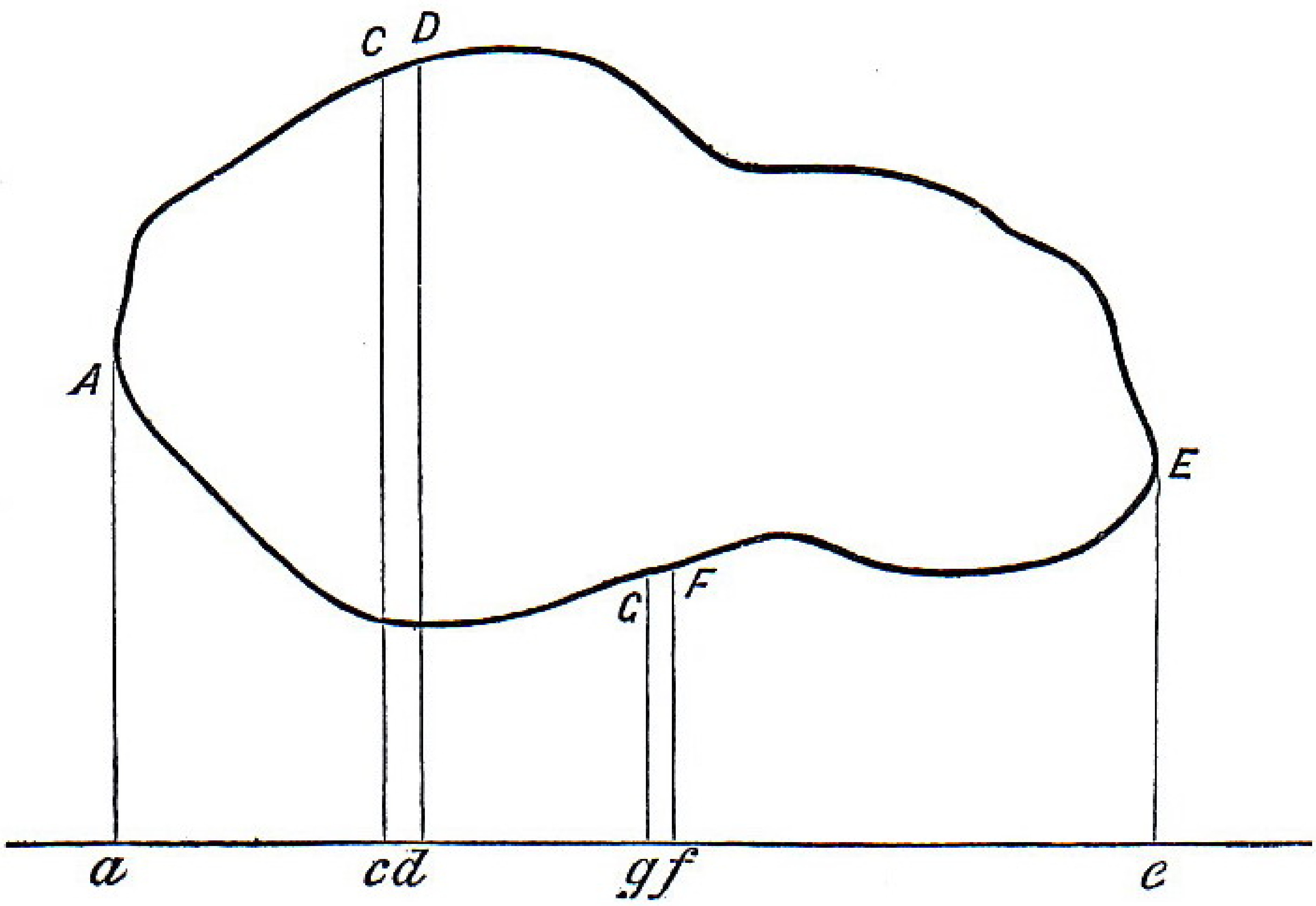
On the same instrument there is an arrangement, by means of bevelled wheels, for changing the direction of the motion of the indices, and thereby converting addition into subtraction.

M. Thomas has provided the means for shifting the addend or subtrahend to a different position on the numeration scale. This he accomplishes by sliding the whole frame carrying the indices sideways, at the same time preserving the recorded number. By this means, when we have set the addend to say 3734, we are able, by shifting the index frame two steps to the right, to convert the addend into 373400. He has also arranged a set of indicators to shew how many times the addition or subtraction has been performed in the various positions, and so to shew the multiplier.

This very ingeniously-constructed instrument exhibits the various characters and requirements of a calculating machine, in a tolerably complete state of development. After all, however, it is only an addition-machine, the only multiplication which it performs being that by the powers of ten, done by the transposition of the frame. A multiplying-machine proper would be such, that when the two factors are set upon the instrument, their product should be exhibited automatically. As yet, we have scarcely any approach to such an instrument.

The first attempt at an apparatus for multiplying was made by John Nepair, the inventor of logarithms; and the study of this attempt may serve to throw considerable light upon the general subject of mechanical aids to calculation. The contrivance is known under the name of Nepair's Bones, and is described in a work called *Rhabdologia.* It is simply a moveable multiplication table. The multiplicand is written on the top of a rod or other moveable slip, its multiples in lines below, the units being separated from the tens by diagonal lines; and these slips are sufficiently numerous to allow of the formation of any number by their initial figures. If the multiplicand be, for example, 43429448, we arrange slips showing that number at the top, and then in any of the horizontal lines we have its multiple by carrying the tens on one slip to the units of that on its right. In this way the computer obtains the successive lines of his product: these he has to write down and add together in the usual way. I am not acquainted with any other contrivance for showing the product of two numerical factors. There exist, however, several instruments for showing the product of factors represented geometrically, and for the summation of such products. It may be useful to examine these, in order to understand the kind of instrument that is desireable for purely arithmetical operations.

In order to obtain the surface of an irregular figure drawn upon paper, we divide it into a number of parts by parallel lines, and get the area of each of these by multiplying the length by the breadth; and finally we add all these products together. The instruments to which I allude perform all of these operations, and show the result at once. In order to understand how this is accomplished, we shall draw a straight line outside of the figure, and bring the parallels to cut it, as in the example. Here we at once perceive that the surface of the figure is the sum of all the rhomboids, such as *c* C D *d*, drawn to the farther side of the contour, less the sum of all such as *f* F G *g* drawn to its inner side.



In order to represent mechanically the areas of such rectangles we must have a motion whose extent is in the compound ratio of the breadth *c d*, and of the length *c* C. In what is, I believe, the first machine of this kind ever contrived, the Platometer of Mr. John Sang, this compounding of two ratios is accomplished by help of a cone, mounted on two wheels. Let us suppose that the axis of the cone is laid in the direction *c* C, and that, in rolling, its apex describes the line *a c*, then while it moves from *c* to *d*, the quantity of turning is proportional to *c d*. But the surface of the cone opposite C D must move through a distance also proportional to *c* C, wherefore, the actual motion there must be proportional to the surface of the rectangle *c* C D *d*. This motion is measured and recorded by a wheel rolling on the cone, and capable of being slid upon it lengthways. Thus when the tracer of the instrument is led along the contour A C D E, the wheel measures and adds up all the minute rectangles from *a* A to E *e*. Similarly when the tracer is led along the other part of the boundary from E to A, the same wheel measures all the rectangles *f* F G *g*, but now in the opposite or subtractive direction; so that when the tracer is brought back to A, the index shows the number of square inches in the surface of the figure.

The Platometer contrived by Mr. Beverley, of Dunedin, New Zealand, exhibits the same principle brought out in a different way. If a cylinder be dragged along a surface endwise it does not turn; if it be moved in a direction at right angles to its axis it rolls; in other positions it partly slides and partly rolls, and in general the quantity of turning is proportional to the sine of the angle between the direction of the axis and the direction of the motion. Taking advantage of this law Mr. Beverley arranges his instrument so that the quantity of angular motion is proportional to the distance of the tracer from a certain straight line; it is, at the same time, proportional to the extent of the actual displacement, and hence the indication is proportional to the area of the rectangle.

These instruments are called Platometers because of the use to which they have been applied; they are in reality integrating machines, and may be applied to a great variety of purposes.

Thus, let us take Mr. Sang's Platometer, and instead of making it to roll like a carriage on wheels, let us fix the frame, leaving the wheels and cone free to turn; and seek to apply it to the solution of such a problem as this, "to find the value of an assurance." We shall divide along the periphery of one of the wheels parts to represent £1; and in the direction of the slope of the cone, other parts to represent the decreasing values of £1 payable $\frac{1}{2}$, $1\frac{1}{2}$, $2\frac{1}{2}$, $3\frac{1}{2}$, $4\frac{1}{2}$ years hence. The product of £1 payable for each person who dies in the year, into the value of £1 for that year is obtained by bringing the index wheel to the proper distance from the apex of the cone, and then turning the cone round by as many divisions as there are deaths in the mortality table. And if we perform this operation regularly for each year during the whole of the table, the index wheel will at once record the sum total of all these products. Beverley's machine may easily be arranged to do the same work.

Both of these instruments, however, and indeed all machines of this kind, depend on a combination of sliding and rolling, and are thus liable to considerable inexactitude in their indications.

They fall far short of the certainty which attends the use of toothed wheels, and would be unfit for such calculations as come under the notice of the actuary.

Having thus given a sketch of the general construction and mode of action of calculating machines, I proceed to consider how these instruments are to be applied and what is the amount of help to be expected from them.

Beginning with the earliest contrivance, that by Nepair, let us place a set of his rods before us, and proceed to make use of them in calculation. We wish, for example, to multiply 397364 by 7. Having collected and arranged the sticks headed 3, 9, 7, 3, 6, 4, we pass down to the seventh row and there find

$$| \;{}^{2}/_{1}\; | \;{}^{6}/_{3}\; | \;{}^{4}/_{9}\; | \;{}^{2}/_{1}\; | \;{}^{4}/_{2}\; | \;{}^{2}/_{8}\; |$$

giving the result 2781548. Now the very least attention is enough to show that the trouble of seeking out the sticks, of arranging them, and of writing down the product, must be many times greater than that of writing down the product without the sticks. It must, not, however, be thought that Nepair's rods were of no use: in his day men learned to count after they were grown old, the Indian numerals were novelties, and the multiplication table was by no means at the finger-ends of computers; thus the rods were then of real use even for a solitary operation such as this.

When we have to find the product of two numbers, each of several digits, the utility of the contrivance is better seen. Thus, if the product of the above number by 782935 were wanted, one arrangement of the rod serves for all the six multipliers, and the six products may be easily written down for addition. There can be no doubt but that this was a considerable aid to those who were unaccustomed to the use of numbers, and on whom the necessity for heavy work had come like an oppression. Astronomy was beginning to assume an exact form, trigonometrical tables had been extended to seven decimal places; the dimensions of the planetary orbits had been ascertained, and even their eccentricities with considerable precision. Hence just as the need for trigonometrical calculations grew greater, the number of decimal places also increased, and the burden of the multiplications and divisions became so intolerable that even Kepler exclaimed, "The duration of human life is too short to meet the requirements of the astronomer." Under such circumstances, the assistance to be obtained from Nepair's rods was by no means to be despised.

Now-a-days, however, an ordinarily good computer, and particularly one who has acquired the easy art of working from the left hand, finds that the time occupied by the mental operation is but an insignificant fraction of that taken up by the manual operation of writing; and to him a set of Nepair's rods would only prove a troublesome incumbrance.

Thus we can only estimate the value of any such mechanical aid, by taking into account the attainments of the operator. To those who can hardly tell whether seven times nine make sixty-five or sixty-three, a set of Nepair's rods may safely be recommended; yet even to such a one the evil may overbalance the good, since he will be prevented from acquiring expertness through practice.

Passing from the elementary contrivance of Nepair to the elaborate and ingeniously contrived machine of Thomas, we have to arrange the indices so as to show the multiplicand, setting at the same time the result indices to zero. On now turning the handle one, two, three times we obtain the product of the multiplicand by the successive digits one, two, three. Having thus obtained the product by the units, we shift the apparatus one step along, turn the handle once for every figure in the tens, so on for the hundreds, until the multiplication be completed: the result has now to be inspected and written down upon our paper. In order to prevent mistake, the multiplier actually used is recorded by the machine, this record has also to be inspected and compared with our data.

The question whether or not this machine manipulation be a help to the computer has to be settled by two considerations; one, the ease of managing the machine, the other, the attainments of the computer himself; it is therefore not susceptible of a general answer.

The transference of the factors from the paper to the machine, the comparisons needed to prevent mistake, and the transference of the result from the machine are serious inconveniences, which appear the greater when we consider that a computer is far more liable to make a mistake in copying than to commit an error in calculation. When we take into account the many short cuts and the abbreviations occurring in general practice, and consider that these are quite lost in the machine work, we begin to think that the balance may not be quite so much in favour of the machine as appears at the first sight of it; and may even be inclined to enquire whether, to an expert computer, the instrument would not prove to be a serious hindrance. And then there is one obvious drawback, that the use of such an aid will not tend to make us good computers.

The application of this machine to division, consists merely in successive subtractions. You by one turn of the handle subtract the divisor, shifted to the proper step on the numeration scale, once from the dividend, and you continue this until the remainder be less than the subtrahend. Having thus discovered the highest digit of the quotient, you shift the dividend one step lower—repeat the subtractions and so continue till the last figure of the quotient appears.

This process contrasts much less favourably than that for multiplying with the ordinary paper work: indeed, to a moderately rapid computer it is irksome, and for this reason that attention has to be given to each successive remainder, lest we should turn the handle once too often; we have, in fact, much more than the mental fatigue of a common division.

The advantages of calculating machines in multiplying or dividing may with some be a matter of doubt; but there can only be one opinion when we apply them to desultory additions or subtractions. We have to add one number to another, or we have to sum up a column of numbers. Having brought the result indices to zero, we set the first addend upon the machine and turn the handle; then set the second addend and give another turn; and so on to the end of the chapter. Why, the time spent in setting the indices would greatly exceed that needed for running up the column and inscribing the result.

The great benefit to be conferred upon us by calculating machines has always been looked for in the compilation of tables; and that benefit is expected in two ways, one in rapidity, and one in certainty of operation.

The simplest kind of table is that of equi-different quantities, such as multiples of the modulus of denary logarithms, the lengths of circular arcs; and for this work the machine seems to be admirably contrived. We have only to put the indices to the common difference, and then each turn of the handle will produce a new term in the series; with a rapidity with which the hand cannot compute, and also with, if the machine be properly made, absolute certainty. True; but where is this result? It is shown on the moveable indices, and, to be of any use to us, must be copied upon paper. Now, the time expended in this copying will be considerably greater than that needed for summing and writing the result in the ordinary way: and although the indication be correct, the copy of it is liable to error; while, if the addition be made continuously on paper, and tested in the usual way, we are almost absolutely freed from error, because an error would necessarily be continued, and so could not escape notice.

In order to obtain the advantage of the certainty of machine action, the machine itself must make its marks upon the paper. Such is the action of numbering machines for railway tickets, for pages of books, and the like. A proper calculating machine for this class of tabular work, punching the results upon a plate, or operating by help of a type-composing machine, would undoubtedly produce trustworthy results.

Nor is the table of equi-different quantities the only one to which machinery may be applied. In our house clocks we have an arrangement whereby changing differences are obtained; these differences increasing by unit each hour. So, by causing the indices of the addend to change at each step, that is, by introducing second differences, we may construct tables of a higher order, such as tables of squares. And again, by employing a third addition table to change the second difference, we may construct tables of cubes. And thus, in general, calculating machines may be made to compute and tabulate the values of all algebraic functions having integer indices. But even such machines are only advantageous when they entirely dispense with pen work.

The great mass of tables, costing labour and thought to the computer, such as trigonometrical canons, tables of logarithms, have their orders of differences interminate, and so it is impossible to compute them by machinery. For then the differences of the last available order must be computed intellectually, and then, indeed, we might set up the type by machinery, provided each successive difference were set by hand. The instrument would therefore fall to be classed among composing machines.

Thus, on the whole, arithmeticians have not much to expect from the aid of calculating machines. A few tables, otherwise very easily made, comprise the whole extent of our expected benefits; and we must fall back upon the wholesome truth that we cannot delegate our intellectual functions, and say to a machine, to a formula, to a rule, or to a dogma, I am too lazy to think, do please think for me.

[The author overlooks the advantage of the arithmometer when used in a very long series of calculations, namely, that the work is almost entirely mechanical, and in consequence much less fatiguing after a very moderate degree of use, than direct calculation, which requires a greater mental strain. This was remarked to us some years ago, by Dr. Zillmer, President of the German Life Insurance Institute, who has used the machine extensively, and who has described some of its applications in a paper of which a translation is printed in the number of this *Journal* for April, 1869, p. 25.—Ed. *J. I. A.*]

Mr. W. J. Hancock, Actuary and Secretary of the Patriotic Assurance Company, who has had much experience in the use of the Arithmometer, has favoured us with the following obsérvations, in further elucidation of the points discussed in the preceding papers of Major-General Hannyngton and Mr. Sang:—

In the foregoing papers by Major-General Hannyngton and Mr. Sang we have a very full description of M. Thomas' arithmometer, but there appears to be a difference of opinion as to its practical utility.

Major-General Hannyngton's recommendation is based on a lengthened practical experience of its working.

Mr. Sang, while stating that on the whole arithmeticians have not much to expect from the aid of calculating machines, does not appear to have used it to any extent.

It is admitted that the machine does the first four rules of arithmetic correctly, and the question is—does it save time, risk of error and mental labour? Take the case of multiplication; we have three methods open to us.

1st. By direct multiplication of the multiplicand by the multiplier, figure by figure.

2nd. By the manipulation of certain other numbers, which bear known relations to the numbers we wish to multiply together, such as logarithms and quarter squares.

3rd. By mechanical aid, such as the slide rule and the arithmometer.

If we take 8 figures to be multiplied by 8 figures—

By the 1st method we have 64 separate multiplications; we have to consider 64 to 72 times what is to be carried, to write down from 64 to 72 figures; then to make 14 additions, some of 8 figures each, and to consider 14 or 15 times what is to be carried.

By the 2nd method the logarithms of the multiplicand must be sought out, and the proportional parts for those figures not given directly in the table used taken out, and the result written down. The same must be done for the multiplier. The numbers so obtained must then be added, and the resulting logarithms converted into the answer required.

By the 3rd method, using the arithmometer, the multiplicand is put on the lower plate or face of the machine; the attention of the operator is then only required to turn the handle the proper number of times, and shift the slide one step for each figure in the multiplier. If on inspection it be found that the figures on the lower plate and on the multiplier disks be correct, the product shown on the upper disks must be correct, unless the machine be broken in some part.

In using the machine we may consider placing the buttons on lower plate as being equal to writing the multiplicand on a piece of paper, and the turning of the handle and shifting the slide to produce the multiplier as a little more than writing the multiplier under the multiplicand. Then we save the mental labour and loss of time incident to long multiplication. The risk of error in the machine is reduced to the risk that some one or more of the figures in the multiplicand and multiplier are wrong. In long multiplication there is, in addition to this risk (a figure may be written on paper wrong as well as be put on the machine wrong), the risk that some of the numerous mental operations to which I have referred may be wrong.

I think it is therefore manifest that the machine is far superior to the long multiplication by hand, when 8 figures by 8 figures are involved, and that it saves mental labour, time and risk of error. Of course the value of the machine diminishes with the number of figures required to be dealt with, and is not perhaps marked when the figures in multiplicand or multiplier do not exceed two.

If a man has not half a mile to travel, there is not much difference between walking and going in a railway train; but when the distance is one or two hundred miles, the advantage of going by train instead of walking becomes evident.

In comparing the machine with the use of logarithms there are two things to be considered:—First, the mental labour and risk of error in finding out the logarithms of multiplicand and multiplier, in their addition, and then in finding the number corresponding to the resulting logarithms. The mental labour here is proportionately greater for the total number of figures used in the operation than in the case of long multiplication. Second, there are many persons who are neither actuaries nor computers who do not know the use of logarithms, and yet have to multiply and divide large figures.

With regard to division the assistance afforded by the machine is, other things equal, only a little less than in the case of multiplication. I cannot agree with Mr. Sang that the mental fatigue of watching the quotient, when working the machine, is greater than in long division. Should the handle be turned once too often, it is at once detected by seeing one or two 9's on the left of the dividend on the slide, and this can be rectified in an instant by putting the machine to addition, when one turn of the handle corrects the error.

The difference between multiplication and division, when performed by logarithms, being only the difference between adding and subtracting the logarithms, the advantage of the machine for division, so far as time and mental labour are concerned, is not quite so great as for multiplication; but, with regard to risk of error, the advantage for both multiplication and division is the same.

Mr. Sang's objections appear to amount to about three.

First. That a computer is far more liable to make a mistake in copying than commit an error in calculation.

Assuming that he is correct in this statement—which I do not admit—I think it is evident that when more than a few figures are dealt with, the computer has far more opportunities of committing an error in calculation than of making a mistake in copying.

Second. That short cuts and abbreviations occurring in general practice are lost in machine work. It is no doubt true that in many classes of cases the quantities to be dealt with bear such a relation to each other that short cuts are of great use. A butcher's boy will sometimes be able to tell how much so many pounds of beef, at so much a pound, will amount to, in a time that would astonish very able computers. Major-General Hannyngton has pointed out in his paper that the machine has short cuts and methods of its own, and that from his experience the machine requires special formulas; and Dr. Zillmer has pointed out the same thing.

The following example of a decreasing annuity calculation will illustrate the use of the machine.

Carlisle $3\frac{1}{2}$ percent (Chisholm):—

(24.) $\frac{(a+b)N_{x+1|n}-b(S_{x+1|n}-nN_{x+1+n})}{D_x}$ = the present value of an annuity for n years, commencing at £a and decreasing by £b annually until the end of the term.

$$x=35,\ x+1=36,\ n=23$$

$$x+1+n=59$$

$a = \begin{cases} 16{\cdot}525 & \text{Instalment} \\ 15{\cdot}203 & \text{Interest on £380·075 at 4 percent} \end{cases}$

	31·728	
b	·661	Interest on instalment at 4 percent
$a+b$	32·389	At 4 percent.

$N_{36}=27615{\cdot}6$
$N_{59}=\ \ 5608{\cdot}3$

$22007{\cdot}3 = \lambda 4{\cdot}3425667$

$32{\cdot}389\ = \lambda 1{\cdot}5103975$

$5{\cdot}8529642 = 712794$

$$
\begin{array}{rl}
S_{36} = & 397165\cdot \\
S_{59} = & 50314\cdot \\
\hline
 & 346851\cdot \\
23N_{59} & 128991 \\
\hline
 & 217860 = \lambda 5\cdot 3381775 \\
 & \cdot 661 \quad \lambda \bar{1}\cdot 8202415 \\
\hline
 & 5\cdot 1583790 = 144005
\end{array}
$$

$$
\begin{array}{rl}
N_{59} & 5608\cdot 3 \\
 & 23 \\
\hline
 & 16824\cdot 9 \\
 & 112166 \\
\hline
 & 128990\cdot 9
\end{array}
$$

$$
\begin{array}{rl}
 & 568789 = \lambda 5\cdot 7549511 \\
D_{35} = 1608\cdot 48 = & \lambda 3\cdot 2064159 \\
\hline
 & 2\cdot 5485352 = 353\cdot 6185.
\end{array}
$$

By machine:—

$$
\begin{array}{l}
\left.\begin{array}{ll} a+b & 32\cdot 389 \\ & \times \\ N_{36}-N_{59} = & 22007\cdot 3 \end{array}\right\} \ldots = 712794 \\
\qquad\qquad\qquad\qquad\qquad\qquad - \\
\left.\begin{array}{l} b \ldots\ldots = \cdot 661 \\ \qquad\qquad \times \\ \left.\begin{array}{l} S_{36}-S_{59} = 346851\cdot \\ \qquad - \\ \left.\begin{array}{l} 23 \\ \times \\ N_{59} \end{array}\right\} = 128991\cdot \end{array}\right\} = 217860 \end{array}\right\} = 144005\cdot
\end{array}
\Bigg\} = 568789
$$

$$
\left.\begin{array}{r} 568789 \\ \div \\ D_{35} \ldots\ldots\ldots\ldots 1608\cdot 48 \end{array}\right\} 353\cdot 6189
$$

The calculation is done as follows:—N_{59} is put on the face of the machine, which is set for multiplication, the handle is turned three times, the slide shifted one step to the right, the handle is then turned twice and 128991 is read from the upper holes and recorded, and the slide effaced. S_{36} is put on the face, the handle turned once and it appears in the upper holes. S_{59} is then put on the face, the machine set for subtraction, and the handle turned once, and 346851 appears in the upper holes. 128991 is now put on the face, the handle turned once, 217860 appears in the upper holes. This number is then put on the face, the slide effaced, the machine set for multiplication, the handle turned once, the slide shifted one step, the handle turned six times, the slide shifted one step, the handle turned six times, and 144005· appears in the upper holes and is recorded. The slide being again effaced, N_{36} is put on the face, the machine set for addition, the handle turned once, and it appears in the upper holes. N_{59} is then put on the face, the machine set for subtraction, the handle turned once, 22007·3 appears in the upper holes. This number is then put on the face, the slide effaced, the machine set for multiplication, the handle turned nine times, the slide shifted one step, the handle turned eight times, and so on, until 32·389 appears on the multiplier disks, then 712794 will appear in the upper holes. 144005 is now put on the face, the machine set for subtraction, the handle turned once, and 568789 appears in the upper holes; the slide can be so placed in the multiplication of $(a+b)N_{x+1|n}$ that these figures will be in the left-hand holes of the slide—otherwise they must be set up in those holes. 1608·48 is then set on the face, the machine set for division, the quotient disks effaced, the handle turned three times, the slide shifted one step, the handle turned five times, and so on until 353·6189 appears in the quotient holes. If the reciprocal of 1608·48 be taken, then 568789 is put on the face, and multiplication is performed until that reciprocal appears in the multiplier holes; then 353·6189 will appear in the upper holes. This calculation is made on a six-figure machine, so that the last figure taken down often has the usual increase of 1 when the next figure is 5 or upwards. The time taken to turn the handle and shift the slide is almost instantaneous. The decimal points are regulated by the ivory pins.

Thirdly. That the use of the machine will not make us good computers.

This objection is exactly of the same nature as the objection often made to the use of machinery in place of hand labour.

In an interesting lecture delivered by the Rev. Professor Haughton, F.T.C.D., M.D., at the Royal Institution of Great Britain, on 27th May, 1871, he points out that the human muscle, like the planet moving in its orbit, or the bee making its cell, performs its work on the principle of least action. I do not see why the brain should not do its work on the same principle; so that if the arithmometer can do those calculations for which it is suited, in less time, or in the same time, but with less mental labour than the hand alone can do them, it should be used. We do not expect the calculating machine to think for us, but to save the brain in doing mechanical work.

During the very interesting discussion which followed the reading of General Hannyngton's paper at the Institute of Actuaries, every gentleman who spoke of having used the machine for any length of time referred to the saving of time and mental labour it effected. Mr. Sprague pointed out that logarithms would be more convenient where three, four, or more factors were to be multiplied together; or where several factors were to be divided by two or three others.

No doubt there are many calculations in which the machine would not give assistance; but that does not make it less useful where it can be applied. A sewing machine is not condemned because it will not sew on buttons. On the whole, I think the balance of argument is in favour of the arithmometer; and I have no doubt that with more extensive use improvements will be made in construction and workmanship.

On the Equitable Apportionment of a Fund between the Life Tenant and the Reversioner. *By* A. Baden, *Fellow of the Institute of Actuaries.*

[Read before the Institute, 27th March, 1871.]

If any excuse be wanting for once more bringing this question forward for your consideration, it is not because the question itself is unimportant. The frequency of the cases involving it upon which the opinion of actuaries is sought, and the magnitude of the